PHYSICS

PAPER—I

Time Allowed : Three Hours | Maximum Marks : 200

QUESTION PAPER SPECIFIC INSTRUCTIONS

Please read each of the following instructions carefully before attempting questions

There are EIGHT questions in all, out of which FIVE are to be attempted.

Question Nos. **1** and **5** are compulsory. Out of the remaining SIX questions, THREE are to be attempted selecting at least ONE question from each of the two Sections A and B.

Attempts of questions shall be counted in sequential order. Unless struck off, attempt of a question shall be counted even if attempted partly. Any page or portion of the page left blank in the Question-cum-Answer Booklet must be clearly struck off.

All questions carry equal marks. The number of marks carried by a question/part is indicated against it.

Unless otherwise mentioned, symbols and notations have their usual standard meanings.

Assume suitable data, if necessary and indicate the same clearly.

Neat sketches may be drawn, wherever required.

Answers must be written in ENGLISH only.

Useful Constants :

Electron charge (e)	=	$1 \cdot 602 \times 10^{-19}$ C
Electron rest mass (m_e)	=	$9 \cdot 109 \times 10^{-31}$ kg
Proton mass (m_p)	=	$1 \cdot 672 \times 10^{-27}$ kg
Vacuum permittivity (ε_0)	=	$8 \cdot 854 \times 10^{-12}$ farad/m
Vacuum permeability (μ_0)	=	$4\pi \times 10^{-7}$ T m/A
Velocity of light in free space (c)	=	3×10^8 m/s
Boltzmann constant (k)	=	$1 \cdot 380 \times 10^{-23}$ J/K
Electronvolt (eV)	=	$1 \cdot 602 \times 10^{-19}$ J
Planck constant (h)	=	$6 \cdot 626 \times 10^{-34}$ J s
Stefan constant (σ)	=	$5 \cdot 67 \times 10^{-8}$ W m^{-2} K^{-4}
Avogadro number (N)	=	$6 \cdot 022 \times 10^{26}$ kmol^{-1}
Gas constant (R)	=	$8 \cdot 31 \times 10^3$ J kmol^{-1} K^{-1}
exp (1)	=	$2 \cdot 718$

SECTION—A

1. (a) A particle of mass m moves under the action of a central force whose potential is $V(r) = Kmr^4$ $(K > 0)$. Calculate the kinetic energy for which the orbit will be circle of radius R, about the origin.

8

   (b) What are the coordinates of the centre of mass of the system of masses shown in the figure?

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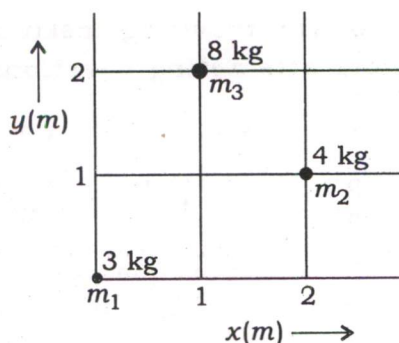
   (c) Calculate the velocity of a particle having kinetic energy four times the rest mass energy.

8

   (d) Explain with proper examples the techniques of obtaining interference by 'division of wavefront' and 'division of amplitude'.

8

   (e) A length of 25 cm of a solution containing 50 gm of solute per litre causes a rotation of the plane of polarization of light by 5°. Find the rotation of the plane of polarization by a length of 75 cm of the solution containing 100 gm of solute per litre.

8

2. (a) A hoop is rolling down on an inclined plane without slipping. Find its velocity at the bottom of the inclined plane.

15

   (b) Calculate the inertia tensor for a rigid body consisting of three particles of masses 3 gm, 1 gm, 2 gm located at $(1, -1, 2)$ cm, $(1, 0, 2)$ cm, $(-2, 1, 0)$ cm respectively.

15

   (c) Define Einstein's coefficients A and B for spontaneous and stimulated emissions, and write the relation between them. If the Einstein's coefficient of spontaneous emission is 10^7/s, determine the Einstein's coefficients for the stimulated emission for the emission wavelength of 5800 Å.

5+5=10

3. *(a)* Consider a thick equiconvex lens of refractive index 1.5 as shown in the figure. The magnitude of the radii of curvatures of the two surfaces is 5 cm. The thickness of the lens is 1 cm and the lens is placed in the air. Obtain the system matrix and determine the focal length and the position of unit planes :
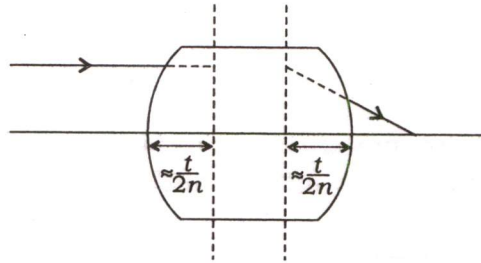
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(b) In a reference frame S, an event 1 occurs at the origin at $t = 0$ and another event 2 occurs at $x = 4000$ m and time $t = 5 \times 10^{-6}$ s. Find the time interval between the events as registered by clock in a frame S' moving with speed $v = 0.6c$ relative to S along the common $X\text{-}X'$ axis, the origin coinciding at $t = t' = 0$.

10

(c) *(i)* Explain an experiment for determination of the refractive index of a gas using Michelson interferometer.

9

(ii) In the above experiment, a shift of 100 fringes is obtained when all the gas is removed from the tube. Calculate the refractive index of the gas if the wavelength of light used is 6000 Å and the length of the tube is 10 cm.

6

4. *(a)* An achromatic doublet of focal length 25 cm is to be made by placing a convex lens of borosilicate crown glass in contact with a diverging lens of dense flint glass. Assuming $n_r = 1.61864$, $n_b = 1.62468$, $n'_r = 1.71426$ and $n'_b = 1.72906$, calculate the focal length of each lens. Here, the unprimed and the primed quantities refer to the borosilicate crown glass and dense flint glass respectively.

15

(b) Find an expression for the average energy of a forced oscillator in the steady-state motion.

10

(c) Explain the formation of spectra by a plane diffraction grating. What particular spectra would be absent if the width of the slit is equal to the width of the opaque spacing between any two consecutive slits?

15

SECTION—B

5. (a) Why does a neutron star have a magnetic field if it is composed of neutrons?   8

   (b) What is the concept of negative temperature in statistical mechanics? Explain in brief.   8

   (c) Explain Ampere's circuital law of magnetic field. What modifications Maxwell made in this equation to derive Maxwell's fourth equation?   8

   (d) Assuming that the Sun radiates like a perfect blackbody, at what wavelength does the peak of the solar spectrum occur? Given that the surface temperature of the Sun is about 6000 K.   8

   (e) If the electrostatic potential in spherical polar coordinate is

   $$\phi(r) = \phi_0 e^{-r/r_0}$$

   where ϕ_0 and r_0 are constants, what will be the charge density at a distance $r = r_0$?   8

6. (a) Distinguish between a perfect gas and a real gas. Derive van der Waals' equation of state and use it to obtain the expressions for the critical constants in terms of the constants of the van der Waals' equation.   15

   (b) Explain how we can produce refrigeration without using a compressor.   10

   (c) Write down Stefan-Boltzmann law of radiation and derive it from Planck's law of radiation.   15

7. (a) Write down the electromagnetic wave equation in non-conducting dielectric medium. Hence, show that the velocity of wave propagation is given by $v = \sqrt{\dfrac{1}{\mu\varepsilon}}$, where the symbols have their usual meanings.   15

   (b) A material has $\sigma = 10^{-2}$ S/m and $\varepsilon = 2\varepsilon_0$. At what frequency, the conduction current would be equal to the displacement current?   10

   (c) An infinitely long coaxial cylinder structure has an inner conductor of radius a and an outer conductor of radius b. The inner conductor is attached to V_0, while the outer one is grounded. Calculate the field $\vec{E}$ in between a and b.   15

8. *(a)* What is Lenz's law? Is Lenz's law the same as Faraday's law? What happens if Lenz's law is reversed? Explain. 15

(b) The wire AB shown in the figure below has mass m, resistance r and it can slide on the smooth horizontal parallel rails separated by a distance l. A uniform magnetic field B exists in the rectangular region and a resistance R connects the rails outside the field region. At $t = 0$, the wire AB is pushed towards right with a speed v_0.
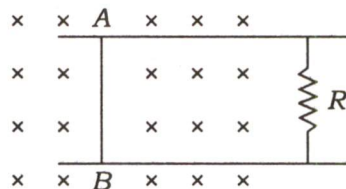
Find out the current in the loop at an instant when the speed of the wire is v. Find out the velocity v as a function of x. (Assume that the resistance of the rails is negligible) 10

(c) Explain how the state of ionization of any particular element in a star changes with varying temperatures and pressures. 15

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